OPTIMIZATION OF ENERGY RELEASE IN A RADIAL HEAT EXCHANGER

V. P. Kolos and V. N. Sorokin

UDC 621.039.517.5

The variational problem of determining the normalized field of energy release, for which the power consumption for pumping the coolant is minimum, is solved for a one-dimensional model of the fluid motion.

The high-potential indicators of heat exchangers of the radial type (the functional diagram is shown in Fig. 1) are based on the combination of the advantages of the energy releasing medium in the form of a layer of monodispersed particles and the advantages of lateral inflow of the coolant [1-4]. Devices of this type can be used in the chemical industry and in power and heat engineering [1-8]. Their extensive possible applications and the expected high efficiency make them the most promising apparatus of this type.

The optimization problem was formulated as follows. For fixed parameters T_0, P_0, and T_L and nominal power from the region of admissable energy release distributions a distribution such that for self-similar profiles of the mass rate of filtering and energy release the power consumption for pumping the coolant is minimum (the dimensions of the channel have an upper limit). Thermodynamic similarity is a condition for optimal heat extraction [4]; it combines inseparably into one problem the problem of searching for optimal energy release and the problem of determining the areas of the distributing and output channels.

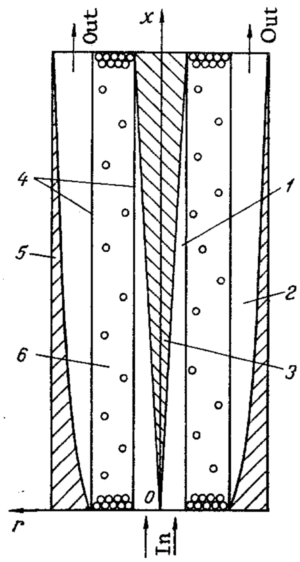

Fig. 1. Functional diagram of a heat exchanger of the radial type with a cylindrical system of coordinates for describing thermohydrodynamic processes: 1, 2) distribution and outflow channels; 3) displacing rod; 4) surrounding gratings; 5) housing (shell); 6) fuel layer.

Institute of Nuclear Power, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 6, pp. 897-905, December, 1987. Original article submitted February 5, 1987.

 0022-0841/87/5306-1374$12.50

In solving the problem we ignore molecular transfer, radiation transfer, the inertial forces of the flow in the layer, heating of the coolant as a result of dissipation, and pressure losses in the enclosing gratings. We shall model the energy release in the particles of the medium by internal heat sources concentrated in the liquid, and we shall assume that it is constant over the thickness of the layer ($q_V = q_V(x)$). We describe the dependences of the coolent density on the thermodynamic parameters by the Boussinesq approximation:

$$\rho = \frac{c}{I}; \quad c = \frac{1}{2}\left[I_{10}(\rho_{10} + \rho_{2L}) + \rho_{2L}\frac{Q}{G_0}\right]. \tag{1}$$

Because the refractive index of the dense fill is an exponential function of its resistance [9] the flow in the fuel layer will be predominantly transverse. Adding to this the fact that the energy release profile need not be calculated with high accuracy, we are fully justified in using one-dimensional equations of thermohydrodynamics for an apparatus of the radial type:

$$P'_{1,2} = -\frac{G}{F}\left(\frac{G}{F\rho}\right)' - \frac{G}{2F^2\rho}G' - \left.\frac{\xi G^2}{2\rho F^2 D}\right|_{1,2}; \tag{2}$$

$$I_1 = I_{10}; \quad I(x;\ r) = I_{10} + 2\pi(G')^{-1}\int_{R_1}^{r} q_V r\,dr; \tag{3}$$

$$I_2 = \frac{\int_0^x I(x;\ R_2)\,G'dx}{G}; \quad G_1 = G_0 - G_2 \overset{\text{def}}{=} G_0 - G;$$

$$\nabla P = -\rho\frac{1{,}7(1-\varepsilon)}{\varepsilon^3 d}|\mathbf{V}|\,\mathbf{V} \Rightarrow \Delta P \overset{\text{def}}{=} (P_1 - P_2) = \frac{1{,}7(1-\varepsilon)}{4\pi\varepsilon^3 d}(G')^2\int_{R_1}^{R_2}\frac{1}{r^2\rho}\,dr. \tag{4}$$

The derivation of these equations is based on the assumption of a strictly radial flow of liquid in the layer [4]. The known quantities are Q, G_0 L, $R_{1,2}$, d, P_0, T_0, $(\Delta/D)_{1,2}$, ε, $B_{1,2}$, N^-, N^+.

We showed previously that for a quadratic law for the resistance of the channels ($\xi_{1,2}$ is independent of the parameters of the flow) in the apparatus the condition of optimal heat extraction is preserved also when the apparatus operates on partial loads. This flow regime is realized in channels with artificial roughness of the wetted surfaces [10]. The coefficients $\xi_{1,2}$ and the minimum value of the coolant flow rate for which self-similarity of the fluid flow in the apparatus is realized [10] were determined for tapered distributing and expanding output channels:

$$\xi_{1,2} = 0{,}145\,(\Delta/D)_{1,2}^{0,322};$$

$$G_0 \geqslant \sup\left(\frac{\mu F}{D}\right)_{10,2L}[10 ** (5{,}27 - 0{,}35\lg 100\,\xi_{1,2})].$$

The further calculations will be based on these formulas.

The optimization problem mathematically reduces to finding the functions $q_l(x)$, $F_{1,2}(x)$ that minimize the functional

$$-\int_0^L\left[\frac{P'_1}{\rho_1}(G_0 - G) + \frac{P'_2}{\rho_2}G + G'\int_{R_1}^{R_2}\frac{1}{\rho}\frac{\partial P}{\partial r}dr\right]dx = \min; \tag{5}$$

with a nonholonomic constraint (the equations of dynamic matching [11])

$$P'_1 - P'_2 - \Delta P' = 0 \tag{6}$$

and the restrictions

$$N^-(x) \leqslant \frac{q_l L}{Q} \leqslant N^+(x); \tag{7}$$

$$\forall x \in [0;\ L),\quad B_1 \geqslant F_1(x) > 0;\quad \forall x \in (0;\ L],\quad B_2 \geqslant F_2(x) > 0;$$
$$G'/G_0 = q_l/Q. \tag{8}$$

With the help of the relation (8) we transform from the unkown function q_ℓ to the quantity $g = G/G_0$ and introduce the longitudinal dimensionless coordinate $x \overset{\text{def}}{=} x/L$. From Eqs. (1)-(4) we determine the power consumption per unit length for pumping the coolant through the layer and the pressure drop ΔP under conditions of thermohydrodynamic similarity (8):

$$G' \int\limits_{R_1}^{R_2} \frac{1}{\rho} \frac{\partial P}{\partial r} = -\varkappa (g')^3;\quad \Delta P = \tilde{\varkappa} (g')^2, \tag{9}$$

where

$$\varkappa = -\frac{0{,}425(1-\varepsilon) G_0^3}{\pi^2 \varepsilon^3 d L^3 c^2} \left[(I_{10} - mR_1^2)^2 \frac{(R_2 - R_1)}{R_1 R_2} + \frac{m^2 (R_2^3 - R_1^3)}{3} + \right.$$
$$\left. + 2m (I_{10} - mR_1^2)(R_2 - R_1) \right];\quad m = \frac{Q}{G_0 (R_2^2 - R_1^2)};$$
$$\tilde{\varkappa} = \frac{0{,}425(1-\varepsilon) G_0^2}{\pi^2 \varepsilon^3 d L^2 c} \left[\frac{(I_{10} - mR_1^2)(R_2 - R_1)}{R_1 R_2} + m (R_2 - R_1) \right].$$

Next we represent the integrand in (5) and the terms in Eqs. (6) explicitly in terms of the unknowns g, $f_{1,2} = F_{1,2}/B_{1,2}$, using for this (1)-(3), (8), and (9). We introduce the penalty functions:

$$f_{1,2}(y_{1,2}) = \frac{1}{\pi} \left(\operatorname{arctg} \frac{y_{1,2}}{\delta} + \frac{\pi}{2} \right); \tag{10}$$

$$g'(z) = N^- + \frac{N^+ - N^-}{\pi} \left(\operatorname{arctg} \frac{z}{\delta} + \frac{\pi}{2} \right) \overset{\text{def}}{=} s(z). \tag{11}$$

They enable avoiding restrictions of the type expressed by the inequalities (7). The variational problem is thereby greatly simplified, in spite of the increase in the number of unknowns and the appearance of an additional nonholonomic constraint (11).

We shall search for g, z, and $y_{1,2}$ amongst the classical set of admissable functions. The linearity of the expressions (5) and (6) as a function of g', z', and $y'_{1,2}$ shows that this set is closed. Here the method of Lagrange multipliers can be used to solve the problem without introducing corrections into Eüler's equations [12]. Following this method, we construct the auxiliary functional, whose integrand will have the form

$$\Phi = -\frac{3(1-g)^2}{2f_1^2} g' - \frac{(1-g)^3}{f_1^3} f_1' + \frac{a_1 (1-g)^3 (1 + \sqrt{1-f_1})}{f_1^3} +$$
$$+ \left(\frac{\rho_1 l}{\rho_2}\right)^2 \left[\frac{3}{2} \frac{g^2}{f_2^2} g' - \frac{g^3}{f_2^3} f_2' + a_2 g^3 \frac{\left(\frac{\sqrt{l}}{n} + \sqrt{\frac{l}{n^2} + f_2} \right)}{f_2^3} \right] +$$
$$+ \mathrm{K} s^3 + \Lambda_1 \left\{ \frac{3(1-g)}{2f_1^2} g' + \frac{(1-g)^2}{f_1^3} f_1' - \frac{a_1 (1-g)^2 (1 + \sqrt{1-f_1})}{f_1^3} + \right.$$
$$+ \left(\frac{\rho_1}{\rho_2}\right) l^2 \left[\frac{3}{2} \frac{g}{f_2^2} g' - \frac{g^2}{f_2^3} f_2' + \frac{a_2 g^2 \left(\frac{\sqrt{l}}{n} + \sqrt{\frac{l}{n^2} + f_2} \right)}{f_2^3} \right] -$$
$$\left. - \tilde{\mathrm{K}} s \left[(N^-)' + \frac{(N^+ - N^-)'}{\pi} \left(\operatorname{arctg} \frac{z}{\delta} + \frac{\pi}{2} \right) + \frac{N^+ - N^-}{\pi} \frac{z'}{\delta + \frac{z^2}{\delta}} \right] \right\} + \Lambda_2 (g' - s); \tag{12}$$

$$K = -\frac{\varkappa \rho_{10}^2 B_1^2 L}{G_0^3}; \quad \tilde{K} = \frac{2\tilde{\varkappa}\rho_{10} B_1^2 L}{G_0^2}; \quad n = \frac{R_1}{R_2};$$

$$a_1 = \frac{\xi_1 L}{4R_1}; \quad a_2 = \frac{\xi_2 L \sqrt{l}}{4R_1}; \quad l = \frac{B_1}{B_2}.$$

Carrying out the standard calculations, we obtain the following system of Euler equations for the function g, z, $f_{1,2}$, Λ_1:

$$\frac{a_1(1-g)(1+\sqrt{1-f_1})}{f_1^3}[2\Lambda_1 - 3(1-g)] + \frac{a_2 g l^2}{f_2^3}\left(\frac{\rho_1}{\rho_2}\right)\left(\frac{\sqrt{l}}{n} + \right.$$
$$\left. + \sqrt{\frac{l}{n^2} + f_2}\right)\left[2\Lambda_1 + 3\left(\frac{\rho_1}{\rho_2}\right)g\right] + \Lambda_1 \frac{1-g}{f_1^3} f_1' + \Lambda_1\left(\frac{\rho_1}{\rho_2}\right)\frac{l^2 g}{f_2^3} f_2' -$$
$$-\Lambda_1'\left[\frac{3(1-g)}{2f_1^2} + \frac{3}{2}\left(\frac{\rho_1}{\rho_2}\right)l^2\frac{g}{f_2^2}\right] = \Lambda_2'; \tag{13}$$

$$\Lambda_1' \tilde{K} s + 3Ks^2 - \Lambda_2 = 0; \tag{14}$$

$$\Lambda_1' - \Lambda_1 \frac{s}{g} + \left(\frac{\rho_1}{\rho_2} g + \Lambda_1\right) a_2 M_2 = 0; \tag{15}$$

$$\Lambda_1' + \Lambda_1 \frac{s}{1-g} + [(1-g) - \Lambda_1] a_1 M_1 = 0; \tag{16}$$

$$(N^-)' + \frac{(N^+ - N^-)'}{\pi}\left(\mathrm{arctg}\frac{z}{\delta} + \frac{\pi}{2}\right) + \frac{N^+ - N^-}{\pi}\frac{z'}{\delta + z^2/\delta} = A + \vartheta_1 f_1' + \vartheta_2 f_2'. \tag{17}$$

Here we employed the notation:

$$M_1 = \frac{1}{f_1}\left[3(1+\sqrt{1-f_1}) + \frac{f_1}{2\sqrt{1-f_1}}\right]; \quad M_2 = \frac{1}{f_2}\left[\frac{f_2}{2\sqrt{l/n^2 + f_2}} - \right.$$
$$\left. -3\left(\frac{\sqrt{l}}{n} + \sqrt{\frac{l}{n^2} + f_2}\right)\right];$$

$$A = \frac{1}{\tilde{K}s}\left\{\frac{3(1-g)}{2f_1^2}s - \frac{a_1(1-g)^2(1+\sqrt{1-f_1})}{f_1^3} + \right.$$
$$\left. + \frac{\rho_1}{\rho_2} l^2\left[\frac{3g}{2f_2^2}s + \frac{a_2 g^2\left(\frac{\sqrt{l}}{n} + \sqrt{\frac{l}{n^2} + f_2}\right)}{f_2^3}\right]\right\};$$

$$\vartheta_1 = \frac{(1-g)^2}{\tilde{K} s f_1^3}; \quad \vartheta_2 = -\frac{\rho_1 l^2 g^2}{\rho_2 \tilde{K} s f_2^3}.$$

We eliminate from Eqs. (13)-(17) the unknowns $\Lambda_{1,2}$ and their derivatives, and we put the system into the form $y_i' = f_i(x; y_1; \ldots; y_N)$, which is convenient for solving on a computer. We shall describe crucial aspects of this mathematical operation. We solve the equations (15) and (16) algebraically for Λ_1 and Λ_1', as a result of which we obtain

$$\Lambda_1 = \frac{\left(\frac{\rho_1}{\rho_2}\right) a_2 M_2 g - a_1 M_1 (1-g)}{\left(\frac{1}{1-g} + \frac{1}{g}\right)s - a_1 M_1 - a_2 M_2} \overset{\mathrm{def}}{=} \sigma; \tag{18}$$

$$\Lambda_1' = -a_1M_1(1-g) + \sigma\left(a_1M_1 - \frac{s}{1-g}\right) \overset{\text{def}}{=} \nu. \tag{19}$$

Differentiating (19) and substituting for g' and z' their expanded expressions (11) and (17), we determine Λ_1'':

$$\Lambda_1'' = R + W_1 f_1' + W_2 f_2', \tag{20}$$

where

$$R = a_1M_1(\nu + s) - \nu\frac{s}{1-g} - \sigma\left[\frac{A}{1-g} + \left(\frac{s}{1-g}\right)^2\right];$$

$$W_1 = a_1Y_1(\sigma - 1 + g) - \frac{\sigma\vartheta_1}{1-g}; \quad W_2 = -\frac{\sigma\vartheta_2}{1-g};$$

$$Y_1 = -\frac{1}{f_1}\left[M_1 + \frac{1}{\sqrt{1-f_1}} - \frac{f_1}{4(1-f_1)^{3/2}}\right].$$

In an analogous manner we find Λ_2' from Eq. (14):

$$\Lambda_2' = \Xi + E_1f_1' + E_2f_2', \tag{21}$$

where $\Xi = R\tilde{K}s + AC$; $E_{1,2} = W_{1,2}\tilde{K}s + \vartheta_{1,2}C$; $C = \nu\tilde{K} + GKs$ (the relations (17) and (20) were employed to derive (21)). After replacing the Lagrange multiplier and the derivatives by (18), (19), and (21) Eq. (13) can be put into the form

$$T + \Theta_1 f_1' + \Theta_2 f_2' = 0. \tag{22}$$

Here

$$T = \frac{a_1(1-g)(1+\sqrt{1-f_1})}{f_1^3}[2\sigma - 3(1-g)] + \left(\frac{\rho_1}{\rho_2}\right) l^2 a_2 g \times$$

$$\times \frac{\left(\frac{\sqrt{l}}{n} + \sqrt{\frac{l}{n^2} + f_2}\right)}{f_2^3}\left(2\sigma + 3\frac{\rho_1}{\rho_2}g\right) - \frac{3}{2}\nu\left(\frac{1-g}{f_1^2} + \frac{\rho_1}{\rho_2}l^2\frac{g}{f_2^2}\right) - \Xi;$$

$$\Theta_1 = \sigma\frac{1-g}{f_1^3} - E_1; \quad \Theta_2 = \sigma l^2\frac{\rho_1 g}{\rho_2 f_2^3} - E_2.$$

Comparing the equalities (18) and (19) we establish a relationship between σ and ν: $\sigma' = \nu$; according to (17)-(19), it can be represented in an expanded form by the linear (with respect to $f_{1,2}'$) equation

$$t + \omega_1 f_1' + \omega_2 f_2' = 0, \tag{23}$$

where

$$t = \frac{1}{\Gamma}\left(\frac{\rho_1}{\rho_2}a_2M_2s + a_1M_1s\right) - \beta\left[A\left(\frac{1}{1-g} + \frac{1}{g}\right) - \left(\frac{s}{1-g}\right)^2 + \right.$$

$$\left. + \left(\frac{s}{g}\right)^2\right] - \nu; \quad \Gamma = \frac{s}{1-g} + \frac{s}{g} - a_1M_1 - a_2M_2; \quad \beta =$$

$$= \frac{1}{\Gamma^2}\left[\frac{\rho_1}{\rho_2}a_2M_2g - a_1M_1(1-g)\right]; \quad \omega_1 = Y_1a_1\left[\beta - \frac{(1-g)}{\Gamma}\right] -$$

$$-\beta\vartheta_1\left(\frac{1}{1-g} + \frac{1}{g}\right); \quad \omega_2 = Y_2a_2\left(\beta + \frac{g}{\Gamma}\frac{\rho_1}{\rho_2}\right) - \beta\vartheta_2\left(\frac{1}{1-g} + \frac{1}{g}\right);$$

$$Y_2 = -\frac{1}{f_2}\left[M_2 + \frac{1}{\sqrt{l/n^2+f_2}} + \frac{f_2}{4\,(l/n^2+f_2)^{3/2}}\right].$$

Keeping in mind the restriction on the range of variation of the solution of the problem (10), the equality (17), (22), and (23) can be easily put into the form

$$y_1' = \pi\left(\delta + \frac{y_1^2}{\delta}\right)\left(\frac{t\Theta_2 - T\omega_2}{\omega_2\Theta_1 - \omega_1\Theta_2} \overset{\text{def}}{=} \gamma_1\right); \quad y_2' = \pi\left(\delta + \frac{y_2^2}{\delta}\right)\left(\frac{t\Theta_1 - T\omega_1}{\omega_1\Theta_2 - \omega_2\Theta_1} \overset{\text{def}}{=} \gamma_2\right); \tag{24}$$

$$z' = \frac{\pi}{N^+ - N^-}\left(\delta + \frac{z^2}{\delta}\right)\left[A + \vartheta_1\gamma_1 + \vartheta_2\gamma_2 - (N^-)' - \frac{(N^+ - N^-)'}{\pi}\left(\operatorname{arctg}\frac{z}{\delta} + \frac{\pi}{2}\right)\right].$$

We shall now formulate the boundary conditions. The property of continuity of the coolant implies

$$g|_{x=0} = 0; \quad g|_{x=1} = 1. \tag{25}$$

We determine the values of the functions $y_1(0)$ and $y_2(1)$ by equating to zero the first variation of the functional:

$$\left.\frac{\partial\Phi}{\partial y_1'}\right|_{x=0} = 0 \Rightarrow y_1(0) = \infty \Rightarrow f_1(0) = 1;$$
$$\left.\frac{\partial\Phi}{\partial y_2'}\right|_{x=1} = 0 \Rightarrow y_2(1) = \infty \Rightarrow f_2(1) = 1; \tag{26}$$

The remaining natural conditions are satisfied for any values of $y_2(0)$, $y_1(1)$ (if (25) holds $\left(\left.\frac{\partial\Phi}{\partial y_1'}\right|_{x=1} \equiv 0,\ \left.\frac{\partial\Phi}{\partial y_2'}\right|_{x=0} \equiv 0\right)$, so that we can assume that they belong to extremals.

The equations (24) and the boundary conditions (26) have singularities at the point $x = 0$ and $x = 1$; the variational problem must be solved on the segment $[\varepsilon; 1-\varepsilon]$, after which we must pass to the limit $\varepsilon \to 0$, using for this purpose spline interpolation [13].

Examining the sufficient conditions for a minimum we note that the extremely $y_{1,2}$, z, and g always satisfy the boundary conditions, and the functional itself is irregular. The fact that the quadratic form $\alpha'\ 1/2\ \Phi''\alpha$ equals zero precludes the use of the Jacobi and Legendre conditions for analyzing the form of the extremum of the functional; new tests are required here. We shall determine the condition under which the sign of the second variation of $\int_2^1 \Phi dx$ is constant. To simplify the notation and for generality of the arguments we represent it in the form

$$\int_0^1 (\Psi + \varphi_l y_l')\,dx,$$

where $\Psi = \Psi(x;\ y)$; $\varphi_l = \varphi_l(x;\ y)$; $y = (\Lambda_{1,2},\ y_{1,2},\ z,\ g) \overset{\text{def}}{=} y = (y_1, \ldots, y_K)$; $l = \overline{1, K}$.

Consider a 3K dimensional vector $\eta = (\eta_1, \ldots, \eta_{3K})$, and let the following relation exist between the comonents of η and y:

$$\eta_l = y_l; \quad \eta_{K+l} = y_l'. \tag{27}$$

We construct the image representation of the auxiliary functional:

$$\int_0^1 (\Psi + \varphi_l y_l') \, dx \Rightarrow \begin{cases} \int_0^1 (\Psi + \varphi_l \eta_{K+l}) \, dx, \\ \eta_{K+l} = y_l' \end{cases} \Rightarrow J \stackrel{\text{def}}{=} \int_0^1 [\Psi + \varphi_l \eta_{K+l} +$$

$$+ \eta_{2K+l} (\eta_l' - \eta_{K+l})] \, dx. \tag{28}$$

Expanding (28) in a Taylor series, we find $\delta^2 J$:

$$\delta^2 J = \int_0^1 \left[\delta_{2K+l,i} \, \delta\eta_l' \, \delta\eta_i + \frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial\eta_i \partial\eta_j} + \frac{\partial^2 \varphi_l}{\partial\eta_i \partial\eta_j} \eta_{K+l} + \frac{\partial \varphi_l}{\partial\eta_i} \delta_{K+l,j} + \right.\right.$$
$$\left.\left. + \frac{\partial\varphi_l}{\partial\eta_j} \delta_{K+l,i} - \delta_{2K+l,i} \, \delta_{K+l,j} - \delta_{2K+l,j} \, \delta_{K+l,i} \right) \right] \delta\eta_i \delta\eta_j \, dx; \quad i, \ j = \overline{1, \ 3K}. \tag{29}$$

In deriving (29) we took into account the fact that $\delta J = 0$; for convenience the indices in the Kronecker δ are separated with a comma. Since the relationship between η_ℓ and $\eta_{K+\ell}$ (see (27)) exists over the entire set of admissible curves (and not only on the extremals) $\delta\eta_\ell' = \delta\eta_{K+\ell}$; the first term in (29) can then be represented as a product of variations of functions:

$$\delta_{2K+l,i} \, \delta\eta_l' \, \delta\eta_i = \delta_{2K+l,i} \, \delta_{K+l,j} \, \delta\eta_i \delta\eta_j. \tag{30}$$

After substituting the expression (30) into (29) and simplifying the result using the properties of a skew-symmetrix matrix, we obtain:

$$\delta^2 J = \int_0^1 \Omega_{ij} \delta\eta_i \delta\eta_j dx,$$

where

$$\Omega_{ij} = \frac{1}{2} \left(\frac{\partial^2 \Psi}{\partial\eta_i d\eta_j} + \frac{\partial^2 \varphi_l}{\partial\eta_i \partial\eta_j} \eta_l' + \frac{\partial\varphi_l}{\partial\eta_i} \delta_{K+l,j} + \frac{\partial\varphi_l}{\partial\eta_j} \delta_{K+l,i} \right),$$
$$i, \ j = \overline{1, \ 2K}.$$

The derivatives η_ℓ' correspond to $y_{1,2}'$, g', z', $\Lambda_{1,2}'$; they are determined from Euler's equations based on the scheme described previously (see (11), (13)-(24)). The existence of uniform convergence makes it possible to determine the form of the extremum with respect to the sign of the symmetric quadratic form $\alpha'\Omega\alpha$: if $\alpha'\Omega_\alpha \geq 0$, then a minimum is realized on the extremal curves; if $\alpha'\Omega\alpha \leq 0$, then a maximum is realized.

The closed system of equations (11) and (24) with the boundary conditions (25) and (26) describes the profile, with the suspicious optimum, of energy release and the throughput sections of the channels, ensuring thermohydrodynamic similarity in the fuel layer; finally, the solution is checked by the test derived.

In conclusion we present some results of a numerical analysis. The starting data are: $N^-(x) = \text{const} < 1$; $N^+(x) = \text{const} > 1$; $\xi_{1,2} \leq 0.07$; the dimensions of the apparatus satisfy the following restrictions: $\frac{0,1L}{R_1} \leqslant 1; \frac{I_{2L}}{I_{10}} \sqrt{\frac{B_1}{B_2}} \sim 1$; $\tilde{K} \geqslant 1$ (the setups are designed to operate under conditions of intense outflow and inflow). The calculations show that for all these constructions the optimal energy release in the layer is practically uniform ($g \approx 1 \rightarrow q_\ell \approx Q/L$); the channel areas are described by the following functions:

$$f_1 \approx (1 - 6a_1)(1 - x)^{3/2} + 4a_1 (1 - x) + 2a_1 (1 - x)^2;$$
$$f_2 \approx \left(1 - \frac{1}{\exp \frac{1}{a_2}} \right) x^{3/2} + \frac{x^{2/3}}{\exp \frac{1}{a_2}}.$$

Naturally, the energy release q_ℓ = const is not always optimal if N^+, $N^- \neq$ const.

NOTATION

x, r, cylindrical coordinates; I, ρ, T, and P, enthalpy, density, temperature, and static pressure of the gas; V, flow velocity; G, flow rate of the gas in the channel; $R_1(R_2)$, inner (outer) radius of the fuel layer; $q_V(q_\ell)$, volume (per unit length) energy release in the layer; Q, thermal power of the apparatus; ε, d, and L, porosity, diameter of the fuel grain, and length of the layer; F, D, ξ, Δ, channel area, the equivalent hydraulic diameter, the coefficient of hydraulic resistance, and the height of the protuberances on the channel walls; $N^-(N^+)$, lower (upper) boundary of the region of admissible distributions of energy release; B is the maximum admissable value of the channel area; δ, an arbitrary positive quantity, close to zero; Λ_1, Λ_2, Lagrange multipliers; $\delta_{i,j}$, Kronecker δ; **, exponentiation; $\overset{\text{def}}{=}$, equality by definition. Indices: 1, parameters referring to the distributing channel; 2, parameters referring to the outflow channel; 1, 2, relations valid for both the distribution and outflow channels; 0, parameters of the coolant at the input; L, parameters at the output; a prime ' indicates differentiation with respect to the longitudinal coordinate.

LITERATURE CITED

1. H. Bartels, Warme und stromumgstechnische Untersuchneng zur Verwendung von directgekuhlten Coated-Particle-Schuttungen in Brennelementen. Ber., Kernforschungsanlage, Julich (1972), No. 824.
2. G. Perona, Energ. Nucl., 11, No. 2, 92-100 (1964).
3. G. Wellovlis, At. Energ. Rubezhom, No. 3, 14-21 (1973).
4. V. P. Kolos, S. M. Kochemazov, and V. N. Sorokin, "Organization of efficient gas motion in a heat exchanger of the poured type," Preprint No. 5, Institute of Nuclear Power, Academy of Sciences of the Belorussian SSR, Minsk (1984).
5. A. I. Khoperskii, At. Energ. Rubezhom, No. 10, 3-13 (1970).
6. S. P. Sergeev and V. V. Dil'man, Khim. Prom-st', No. 8, 469-470 (1982).
7. E. P. Sheludyakov and I. E. Chernyakov, Improvement of the Technology and Organization of Grain Harvesting and Post-Harvest Technology Technology, Siberian Branch of the V. I. Lenin All-Union Academy of Argicultural Sciences, Novosibirsk (1983), pp. 135-142.
8. I. E. Idel'chik, Aerodynamics of Commercial Equipment [in Russian], Moscow (1964).
9. A. M. Vaisman and M. A. Gol'dshtik, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 4, 74-80 (1978).
10. V. P. Kolos and V. N. Sorokin, Dokl. Akad. Nauk Nauk BSSR, 25, No. 6, 526-529 (1981).
11. A. E. Pobedrya, V. I. Khorev, and V. P. Kolos, Vestsi AN BSSR, Ser. Fiz. Energ. Navuk, No. 2, 21-26 (1976).
12. L. Yang, Lectures on Variational Calculus and the Theory of Optimal Control [in Russian], Moscow (1974).
13. G. Forsythe, M. Malcolm, and K. Moler, Computer Methods for Mathematical Computations [Russian translation], Moscow (1980).